

IPM component 3 – STEM

On being an expert

Expert opinion

How do you tell the difference between an expert and a non-expert in your discipline? What qualifies as expert opinion and how do you recognise it? Let us start with some examples.

Physics example

The following is an excerpt from a professional scientist, Robert Boyle (1627 – 1691), trying to explain in 1660 the idea of the springiness of air (which today we call “pressure” and relates pressure of a gas to the volume of the container the gas is in):

“Of the structure of the elastic particles of the air, divers conceptions may be framed, according to several contrivances men may devise to answer the phenomena: for one may think them to be like the springs of watches, coiled up, and still endeavouring to fly abroad. One may also fancy a portion of air to be like a lock or parcel of curled hairs of wool; which being compressed ... may have a continual endeavour to stretch themselves out, and thrust away the neighbouring particles [...]

Only I shall here intimate, that thought the elastic seem to continue such, rather upon the score of its structure, than any external agitation; yet heat, that is a kind of motion, may make the agitated particles strive to recede further and further .. and to beat off those, that would hinder the freedom of their gyrations, and so very much add to the endeavour of such air to expand itself.”

(taken from *Words, Science and Learning*, Clive Sutton, p75)

Mathematics example: Mathematical terminology

Supposing a member of the public comes to a lecture on mathematics. How would I distinguish between their understanding of terms such as “average” or “infinity” from the mathematician’s understanding of these terms? As the “expert” I would have to know the audience’s general understanding of these terms and clarify the difference between this and the correct meaning of these terms:

1. "Average" = average (general audience)
= mean, mode, median (expert audience)
 - The mean is the usual average of a list of numbers. I.e. it is the sum of all the numbers in a list, divided by the number of numbers in that list.
 - The mode is the value that appears most often in a list of numbers.
 - The median is the number which separates the higher half of a list of numbers from the lower half of that list. i.e. it is the middle number of a list of numbers when that list is put in ascending order.

2. "Infinity" = a very big number (general audience)
= something which has no finite size; something which is undefined; something which cannot be counted in a finite number of steps.
(expert audience)

3. "Theory" = speculation; an opinion, where your opinion is as good as mine, and our opinions can be different (general audience)
= a conclusion arrived at in a finite number of mathematical steps, each step being logically correct from the previous step according to the rules of mathematics. A theory is factually correct and cannot be disproved.
(expert audience)

4. "Solution" = the answer to a problem (general audience)
= all the necessary mathematical steps needed, to get from the question to the final answer, this latter being the very last step of the solution.
(expert audience)

5. "Dimension" = size (general audience)
= the number of independent (orthogonal) axes. E.g. n -dimensional space.
(expert audience)

6. “Integer” = a number (general audience)
 = integer, as opposed to rational numbers, real numbers or complex numbers. (expert audience)

Physics/engineering example: Technical terminology

Consider the text below which is taken from *Mechanical Vibrations* by S. S. Rao:

“If a force $F(t)$ acts on a viscously damped spring-mass system [...], the equation of motion can be obtained using Newton’s second law [...].”

Supposing a member of the general public comes to a public lecture on the above subject of mechanical vibrations. How would you distinguish between their understanding of the terms

Force	Viscosity	Damped	Spring
Mass	System	Equation	Motion

and the expert’s understanding? How do you bridge the gap in the way general audience would understand the scientific meaning of these terms from the way experts would understand these terms?

Exercise

Fill in the table below.

Term	General audience’s understanding (Lay meaning)	Expert audience’s understanding (Scientific meaning)
Average		
Dimension		
Differentiate		
Water		

Atom		
Cohesion		
Adhesion		
Acid		
Force		
Thermometer		
Viscosity		
Damping		

Physics/engineering example: Describing a technical text in non-technical language

Consider the text below which is taken from *Mechanical Vibrations* by S. S. Rao:

“If a force $F(t)$ acts on a viscously damped spring-mass system [...], the equation of motion can be obtained using Newton’s second law [...].”

My attempt at describing the meaning of this text is

Suppose you are driving along a road which has potholes in it, and you drive over one of these potholes. Some shock absorbers tend to have two aspects which cushion the reaction of the car driving over the pothole. One is a coiled spring made of some suitable material (metal, carbon fibre, or other), and the other is some kind of oil inside the cylinder part of the shock absorber.

Now, suppose the coiled spring is made of metal. The type of metal used, and the number of coils in the spring, will determine how springy the shock absorber is. Using one type of material and/or a certain number of coils, and you will get a soft,

spongy reaction when you drive over the pothole. This means that after you have driven over the pothole you could keep bouncing down the road for ages.

Using another type of metal, and a different number of coils in the spring, may mean that there is no bounce at all as you drive over the pothole. It is as if you had no suspension at all on the car, and this would make for very uncomfortable drive.

The viscosity of this oil relates to how “thick” it is. The more viscous it is, the “thicker” it is, and therefore the “harder” it reacts against impact, the more quickly it acts to absorb or stop any rebound, and the more quickly it will stop the car bouncing up and down. This then makes it less comfortable for the driver.

The less viscous the oil is, the less quickly it acts to absorb or stop rebound, and the less quickly it will stop the car bouncing up and down. In other words the car will bounce up and down for longer, and will make for a more spongy feeling for the driver.

The degree of viscosity of the oil then determines the degree of springiness of the oil. Taken together, the coiled spring and the oil are considered as one single spring-mass system, this having one overall, total, springiness.

We want to study the way the car bounces up and down as a result of the type of shock absorber used. The equations which relating to the car’s bouncing up and down can be found from Newton’s second law of motion. This says that when a force acts on an object, the object accelerates in the direction of the force. If the mass of an object is held constant, increasing force will increase acceleration.

Here the fall of the car into the pothole is the acceleration being referred to. What force causes this acceleration? Gravity, pulling the car down into the pothole. Since gravity is (more or less) constant, the heavier the car, the more it will affect driving comfort.

Exercises

1) Now compare the difference between the original text and my description in terms of expert knowledge, language and communication, and the knowledge of a novice.

2) In-class discussion: Using the two examples above as reference list some of the qualities and traits you believe the expert solver has which a novice would not have.

Mathematics example: On the finding the roots a quadratic equation

There are four ways of solving $ax^2 + bx + c = 0$. The one you choose depends upon the “complexity” of the quadratic, namely the way in which the coefficients interact arithmetically:

	Direct factoring	Direct square rooting	Completing the square	Quadratic formula
Archetypal equation	$x^2 + 4x = 0$	$x^2 - 25 = 0$	$x^2 - 4x - 32 = 0$	$ax^2 + bx + c = 0$ for $b^2 \geq 4ac$
Advantage	Fast and easy to do;	Fast and easy to do;	Works for all types of quadratics.	Works for all types of quadratics.
Disadvantage	Only works when there is no constant term	Only works when there is no term in x	Algebra/arithmatic can be complicated?	Algebra/arithmatic can be complicated?

Mathematics example: Algebraic closure or not

Consider wanting to find the roots of the following polynomial: $y = x^2 + 3x + 2$. This means we want to solve $x^2 + 3x + 2 = 0$. If we only want to work in integers, then it is possible to solve this equation. Doing so we obtain $x = -1$ and $x = -2$. But what integer values of x solve $6x^2 - x - 1 = 0$? There are no integer values which make this work. In fact, the answers are $x = \frac{1}{2}$ and $x = -\frac{1}{3}$.

So polynomial equations with integer coefficients do not always give you integer roots, since in this last case we have rational roots. This is an example of algebra not being closed under integer arithmetic.

So, do polynomial equations with rational coefficients always give you rational roots? No. The polynomial $\frac{1}{2}x^2 - 1 = 0$ has a rational coefficient (i.e. the $\frac{1}{2}$) but its solution is $x = \sqrt{2}$ which is irrational.

So polynomial equations with rational coefficients do not always give you rational roots, since in this last case we have an irrational root. This is an example of algebra not being closed under rational arithmetic.

So, do polynomial equations with irrational coefficients always give you irrational roots? No. The polynomial $\frac{1}{2}x^2 + \frac{3}{4}x + 5 = 0$ has irrational coefficients (i.e. the $\frac{1}{2}$ and the $\frac{3}{4}$) but its solution gives complex numbers.

So polynomial equations with irrational coefficients do not always give you irrational roots, since in this last case we have complex numbers. This is an example of algebra not being closed under irrational/real arithmetic.

Finally, do polynomial equations with complex numbers as coefficients always give you complex numbers as roots? Yes. The polynomial $ix^2 + (2 - 3i)x + 5 = 0$ has complex coefficients (i.e. the i and the $2 - 3i$), and its solution will give complex numbers. This is an example of algebra being closed under complex arithmetic.

Commentary

In terms of mathematics example 1 above we might say that an expert can recognise the different types of quadratics, and know not only how to use each root-finding method on selected quadratics but also know which method is best to use under which circumstance. Specifically, s/he can look at the quadratic and recognise its structure, which will then lead him/her to choose one method technique over another. The expert also knows the strengths and weaknesses of each technique and therefore understands the need for so many different techniques. The expert is then able to present the solution to the finding the roots of a quadratic in a logical and coherent manner. All of this (and more) leads to the expert to being able to think in a “quadratic equation” manner.

More generally, an expert has more depth and breadth of knowledge about a subject than a novice. S/he knows how to investigate their subject whereas a novice will (probably) not know this. This leads the expert to know when to use different aspects of his/her discipline, i.e in what situations s/he needs to use this-or-that technique or mathematical theorem. S/he is able to explain what steps were done at each stage of the solution and why each step was performed in this-or-that particular way.

Allied to this is the fact that experts have a degree of theorisation, abstraction and generalisation about their subject that novices do not have. And they also use highly technical methods of analysis: mathematical reasoning, reasoning about the physics of the problem, technical diagrams, experiments, computer programs and simulations, etc.

Beyond just methods and techniques experts use specific methodologies or protocols to study things in a structured, systematic way. For example, scientists must systematically collect, analyse, and interpret data about the natural world and do so according to standards of rigor, relevance. Novices don't have such an approach.

But before the expert can do this s/he must have a question to answer or a problem to solve. Experts are capable of “[...] identifying and framing a meaningful and productive question for investigation based on the existing state of knowledge in the researcher's discipline, formulating a testable research hypothesis based on a specific question, designing a valid experiment or empirical test of the hypothesis, and interpreting data by relating results to the original hypothesis and drawing appropriate, supportable conclusions (Kardash, 2000). Collectively, these skills result in the construction of disciplinary arguments within a scientific discipline, the mastery of which is considered essential for successful scientists.” (taken from *The Development of Expertise in Scientific Research*, David Frank Feldon, October 2017, www.researchgat.net, DOI: 10.1002/9781118900772.etrds0411).

The research question then leads to another difference between experts and novices: experts study a much narrower part of a topic compared to novices. They do this in order to make the problem manageable or solvable so as to get an initial solution to the problem. For example, I once studied the motion of a single electron (not even two electrons!) travelling in the Earth's magnetic field under the assumption that no other external force was affecting the motion. This is a very specific thing to be studying.

Experts can be wrong:

1. *Astronomy*: Ptolemy's Earth-centred planetary system versus Copernicus' sun-centred planetary system;
2. *Chemistry*: Phlogiston which was supposed to explain an aspect of combustion (until the element Oxygen was discovered);

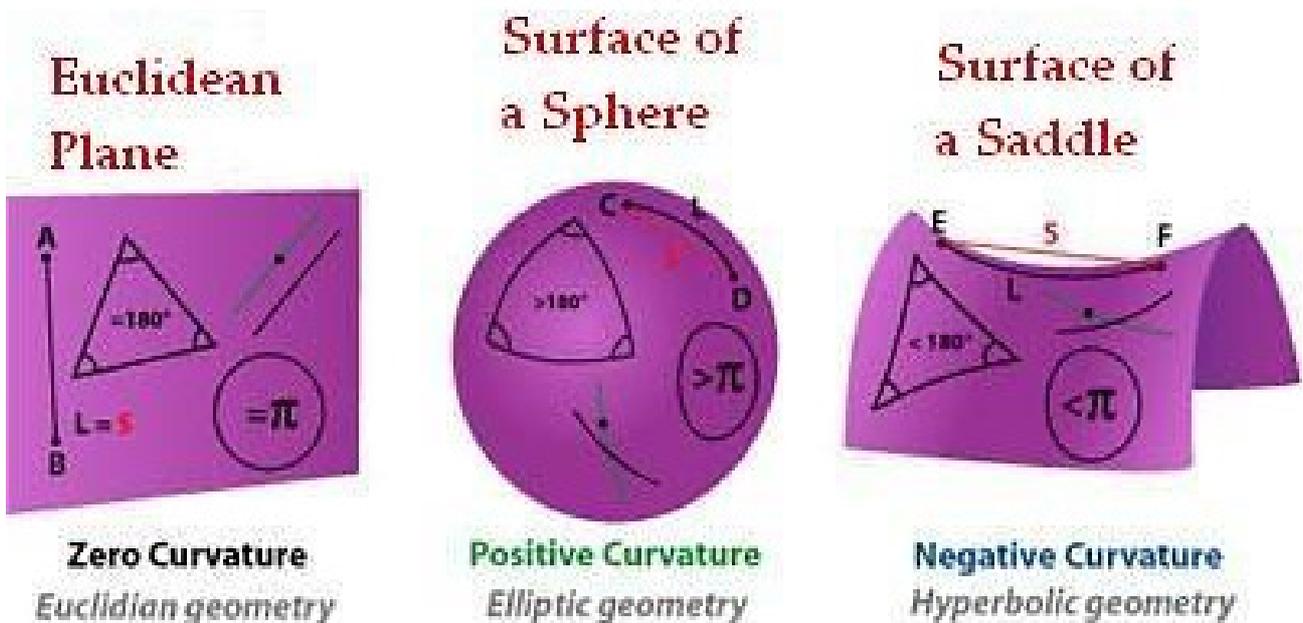
3. *Physics: The smallest unit of matter is the atom (until people started finding protons, neutrons and electrons, quarks, leptons, etc.);*
4. *Physics: The nature of heat:* Heat was originally conceived of as a property inherent within matter. In other words metal was cold because that was an internal property of metal. As science progressed heat came to be understood as a transfer of energy. In that case things could be “heated up”, and metal could be hot if enough energy was applied to it from an external source (fire). The “coldness” of metal could now be seen as a specific energy transfer at a specific temperature and pressure.
5. *Physics: The nature of force:* Force also was originally conceived of as an internal property of matter. In other words, object inherently wanted to move or remain stationary. This was part of the nature of an object.

“Aristotle believed that objects intrinsically either have a natural tendency to fall down to the earth, which he called gravity, or a natural tendency to rise into the sky, which he called levity. He thought that heavy bodies fall faster because the falling speed is in proportion to the physis (nature) or weight of the objects. The earth and the sky are natural places objects would move to according to their internal natural tendencies.”

(from <http://www.thecatalyst.org/physics/chapter-two.html>)

6. *Mathematics:* Multiplication is always commutative, i.e. $a \times b = b \times a$ for all numbers, until people discovered this is not true for matrices and for the vector product.
7. *Mathematics:* In the days of the ancient Greeks, right up to 17th – 18th centuries, it was believed that negative numbers could not exist because you could not have a negative size or length or weight. Only positive numbers existed because the concept of a positive number was always related to the physical world (until people discovered that algebra produced answers which were negative (or even complex), and that it was possible to develop a coherent mathematical framework of numbers from the whole numbers \mathbb{N} to the integers \mathbb{Z} to the rationals \mathbb{Q} to the reals \mathbb{R} to the complex \mathbb{C} .

8. *Mathematics*: Up until the mid-1800s it was believed that Euclidean geometry (the geometry of the plane) was the only geometry possible for mathematics. Then two Bolyai (1802 – 1860) and Lobatchevsky (1792 – 1856) separately developed non-Euclidean geometry, specifically hyperbolic geometry, by which a consistent mathematics could also be developed.



(from http://www.huffingtonpost.com/mauricio-garrido/lessons-from-non-euclidian-geometries-for-interfaith-dialogue_b_3403930.html)

This last example brings up the next topic which we will address, namely absolutism and relativism. From the time of Euclid (~300BC) to the late 1800s Euclidean (plane) geometry was thought to be the only geometry. This geometry was also thought to be the foundation of mathematics. Everything in mathematics could be expressed in geometric terms.

Then during the 1800s the two mathematicians mentioned above, Lobachevsky and Bolyai discovered another form of geometry that was equally mathematically valid and correct. This was spherical geometry, as illustrated by the middle image in the diagram above. Here geometry is performed on a spherical surface as opposed to a flat plane (as in Euclidean geometry). In this case it is found that the sum of angles in a triangle is greater than 180° and lines which start off parallel end up intersecting. There is also yet another type of geometry called hyperbolic geometry (better described as the geometry of a saddle) whereby the sum of angles in a triangle is less than 180° and lines which start off parallel end up diverging.

So, the idea of the sum of angles in a triangle or the idea of parallelism is dependent on the type of geometry we are using. In that case there is no such thing as an absolute geometry. There is no such thing as the one and only geometry from which all of mathematics is derived, because we could ask, Which geometry do I use to derive my mathematics?

See separate notes for details on the topic of absolutism and relativism.

Exercises

1. Who were/are considered experts in your field? What field were/are they expert in?
2. Look at the history of your discipline. Were there any theories which were believed true but later turned out to be incorrect? If so which ones?